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Diffraction-Free Beams

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It was recently predicted that nondiffracting beams, with beam spots as small as a few wavelengths, can exist and propagate in free space. We report the first experimental investigation of these beams.

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Diffraction is one of the universal phenomena of physics, and one of the best understood. It affects all classical wave fields without exception. Furthermore, given the de Broglie postulate associating particle momentum inversely with wavelength, diffraction is the fundamental basis for quantum-mechanical uncertainty relations, as Heisenberg explained very clearly.

It is the Helmholtz equation that governs diffractive phenomena in every area of physics:

$$[\nabla^2+\kappa^2]\Phi(r;\kappa)=0. \tag{1}$$

However, Durnin[1] has recently pointed out that the Helmholtz equation has a class of *diffraction-free mode solutions.* One recalls that techniques for reducing diffractive spreading and enhancing beam directivity have long been pursued, for example, in the design of super-gain antennas. Toraldo di Francia[2] gives a concise discussion of the intrinsic limitations of these techniques, and makes clear that in any event they aim for a reduction, not an elimination, of diffractive spreading.

Of course, plane waves are diffraction-free mode solutions of the Helmholtz equation, but Durnin's diffraction-free modes can have the startling property that they describe well defined beams with narrow beam radii. *The central spot radius can be extremely narrow, on the order of one wavelength, without being subject to diffractive spreading.*

The simplest of Durnin's predicted nonspreading beam solutions, which is easily shown to satisfy (1), is a monochromatic wave propagating in the z direction with field amplitude

$$\Phi(x,y,z;\kappa)=\exp[i\beta z]J_0(\alpha\rho), \tag{2}$$

where $\alpha^2+\beta^2=\kappa^2$, $x^2+y^2=\rho^2$, and J_0 is the zeroth-order Bessel function of the first kind. When $0<\alpha<\kappa$, solution (2) represents a *nondiffracting beam* because it has the same intensity distribution $J_0^2(\alpha\rho)$ in every plane normal to the z axis. The half-width of the central peak is approximately α^{-1}, and the transverse skirt of the distribution decays as ρ^{-1}.

It must be emphasized that these are exact, nonsingular solutions appropriate to free space (no boundaries or guiding surfaces or nonlinear media) and they are not the packet-type solutions recently discussed by Brittingham,[3] Belanger,[4] and Ziolkowski.[5]

The detailed properties of the beam associated with solution (2) have been described elsewhere[1] and we summarize the most important of these properties in Fig. 1. The figure shows the transverse intensity profile of the J_0 beam and of a Gaussian beam with the same spot size at $z=0$. Normal diffractive beam spreading, accompanied by rapid decay of peak intensity, is exhibited by the

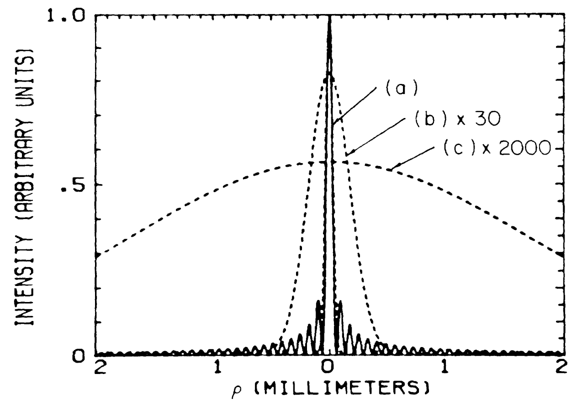

FIG. 1. The transverse profiles of a J_0 beam and of a Gaussian beam with the same spot size (FWHM = 70 μm) in the $z=0$ plane are shown at a succession of z values. The J_0 beam is invariant to propagation along the z axis, but the Gaussian beam exhibits normal diffractive spreading and a rapid decrease of peak intensity. The dotted curves indicate the Gaussian beam at the distances: Curve a $z=0$, curve b $z=10$ cm, and curve c $z=100$ cm, if we assume $\lambda=6328$ Å. The Gaussian intensities at $z=10$ and 100 cm have been multiplied by factors of 30 and 2000, respectively, to permit visibility.

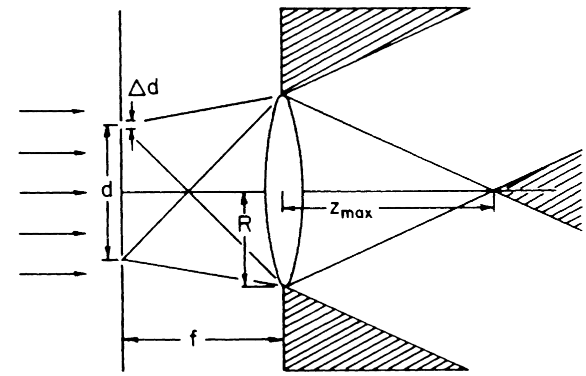

FIG. 2. Experimental arrangement for the creation of a J_0 beam. Collimated light of wavelength λ illuminates a circular slit located in the focal plane of a lens. The mean diameter of the slit is d, the width of the slit is Δd, the focal length of the lens is f, and the radius of the output aperture is R. The distance $Z_{\max}$ indicates the beginning of the geometrical shadow zone along the z axis.

Gaussian beam, while the J_0 beam has the same intensity profile at every value of z. Of course, Durnin's solutions are rigorously exact only in infinite free space, whereas any realizations of such beams in a laboratory will necessarily be limited by a finite aperture.

In this note we report the first experimental investigation of Durnin's nondiffracting beams. A beam having the same parameters of Fig. 1 was created by means of the setup shown in Fig. 2. A circular slit of mean diameter $d=2.5$ mm and width $\Delta d=10$ μm was placed in the focal plane of a lens of focal length $f=305$ mm and radius $R=3.5$ mm. Ideally, each point along the slit acts as a point source which the lens transforms into a plane wave. It is not hard to see that the set of plane waves formed in this way has wave vectors lying on the surface of a cone, and Durnin has shown that this can be regarded as the defining characteristic of the J_0 beam. When the slit is illuminated with collimated light of wavelength λ, one then obtains a J_0 beam with spot parameter $\alpha=(2\pi/\lambda)\sin\theta$, where $\theta=\tan^{-1}(d/2f)$. In practice, of course, the amplitude is modulated by the diffraction envelope of the slit. That modulation is negligible within the finite output aperture R, provided that $\Delta d \ll \lambda f/R$, as it is in this particular case.

In our experiment the radius of the lens shown in Fig. 2 defines the finite aperture. According to geometrical optics, a shadow zone begins along the z axis at a distance $Z_{\max}=R/\tan\theta$ from the aperture. Since the beam radius is approximately $r=\alpha^{-1}$, we can use $\tan\theta \approx \sin\theta=\alpha/\kappa=\alpha\lambda/2\pi$ to express the maximum propagation distance as $Z_{\max}=2\pi Rr/\lambda$.

This geometrical estimate of the maximum range should be compared to the distance that an ordinarily collimated or apodized beam can propagate without significant spreading. The usual distance over which a beam of radius r remains transversely well localized while propagating in free space is the Rayleigh range $Z_r=\pi r^2/\lambda$. Thus the propagation range predicted from Fig. 2 will be much larger than the conventional range of a beam of radius α^{-1} whenever $R \gg \alpha^{-1}$.

In Fig. 3(a) we show a numerical simulation of the propagation of the peak intensity of both the Gaussian and the J_0 beam shown in Fig. 1, taking into account the finite initial aperture diameter $2R=7$ mm. The peak intensity of the Gaussian beam decreases by an order of magnitude after propagating only 5 cm. The peak intensity of the J_0 beam, on the other hand, oscillates about its initial value with increasing amplitude and decreasing frequency until reaching a point where a sharp decline occurs. (Since these intensity oscillations are reminiscent of the Fresnel diffraction pattern near a knife edge, it is important to note that this graph represents the propagation of peak intensity away from the aperture rather than diffraction in the transverse plane near the aperture.) Using the numerical values given above, one finds that the geometrical estimate $Z_{\max}$ for the maximum range[6] of the J_0 beam is 85 cm, a point located exactly at the base of the sharp decline in peak intensity shown in Fig. 3.

In Fig. 3(b) we show the curve of Fig. 3(a) with experimental data points superimposed. A J_0 beam having the parameters given above was measured every centimeter from $z=10$ cm (the outer surface of our lens) to $z=1$ m. The data are well fitted by the numerical simulation of the propagation of peak intensity. We have also observed the transverse profile of the beam as a function

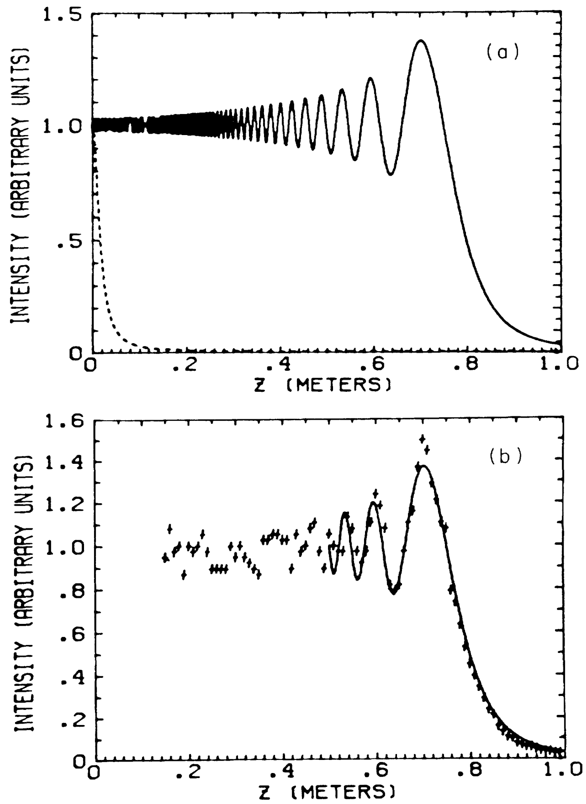

FIG. 3. (a) Numerical simulation of the on-axis intensity of the J_0 and Gaussian beams shown in Fig. 1, taking into account the finite aperture of the lens used to create the J_0 beam. The dashed curve shows the rapid decrease in on-axis intensity associated with normal diffractive spreading of the Gaussian beam. (b) Same curve as in (a) with experimental data added (and curve removed for $z < 50$ cm to allow visibility of data points).

of distance z from the aperture and found that the predictions implied by Fig. 1 are also obeyed. No measurable spreading of the central peak profile could be observed over a propagation distance of almost 1 m. A comprehensive report on our experimental results is currently in preparation.

In conclusion, we have observed the most important properties of the ideal J_0 beam. We have confirmed that beams exist whose central maxima are remarkably resistant to the diffractive spreading commonly associated with all wave propagation. This finding is not in conflict with Heisenberg's uncertainty relation $\Delta p \Delta x \geq \hbar/2$ or the laws of diffraction. The reason is that, for a J_0 beam, Δx is actually infinite because of the ρ^{-1} transverse dependence of $J_0^2(\alpha\rho)$ for large ρ. Despite its infinite rms radius, however, it is clear that the J_0 beam has a well defined sharply peaked radial profile. Finally, we have also shown that a laboratory realization of these beams is feasible using only simple optical elements.

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[6]In fact, from numerical simulations it has been found that the geometrically estimated range of propagation coincides with the effective nonspreading range of J_0 beams of finite aperture for all values of α between $\alpha = 2\pi/\lambda$ (when the wave is evanescent and $Z_{max} = 0$) to $\alpha = 2\pi/R$ (when the source field is essentially just a uniform disk of radius R).